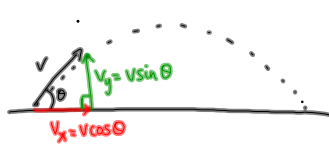


Symmetrical Trajectories (A projectile that returns to the same level)

A special case since the vertical displacement is ZERO!



Find expressions for:

- a) Δt
- b) Δx
- c) H (max height)

a) To find the time the projectile is in the air, then we need to look at the vertical motion.

vertically
 $v_i = v \sin \theta$
 $a = -g$
 $\Delta d = 0$
 $\Delta t = ?$

$$\Delta d = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$0 = v \Delta t \sin \theta - \frac{g}{2} (\Delta t)^2$$

$$0 = \Delta t (v \sin \theta - \frac{g}{2} \Delta t)$$

$$\Delta t = 0 \quad v \sin \theta - \frac{g}{2} \Delta t = 0$$

$$v \sin \theta = \frac{g}{2} \Delta t$$

$$\Delta t = \frac{2v \sin \theta}{g}$$

b) To find the horizontal displacement, we need to use the horizontal velocity and the Δt .

Horizontally $v_x = \frac{\Delta d_x}{\Delta t}$

$$\Delta d_x = v_x \Delta t$$

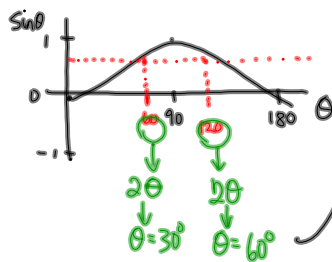
$$\Delta d_x = (v \cos \theta) \left(\frac{2v \sin \theta}{g} \right)$$

$$\Delta d_x = \frac{v^2 (2 \cos \theta \sin \theta)}{g} \rightarrow 2 \cos \theta \sin \theta = \sin 2\theta$$

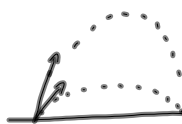
(trig ident)

Range of projectile $\rightarrow \Delta d_x = \frac{v^2 \sin 2\theta}{g}$

A launch angle of 45° gives you the maximum range



A launch angle 30° and 60° will have the same



c) Maximum height:

$$\Delta d_y = ? \text{ (max height)}$$

$$a = -g$$

$$v_i = v \sin \theta$$

$$v_2 = 0 \text{ (at max height)}$$

(complementary launch angles give the same range)

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$0 = v^2 \sin^2 \theta + 2$$

$$2g\Delta d_y = v^2 \sin^2 \theta$$

max height $\rightarrow \Delta d_y = \frac{v^2 \sin^2 \theta}{2g}$